深圳鸿芯微纳技术有限公司
Shenzhen Giga Design Automation

Asymptotic Waveform Evaluation for Timing Analysis

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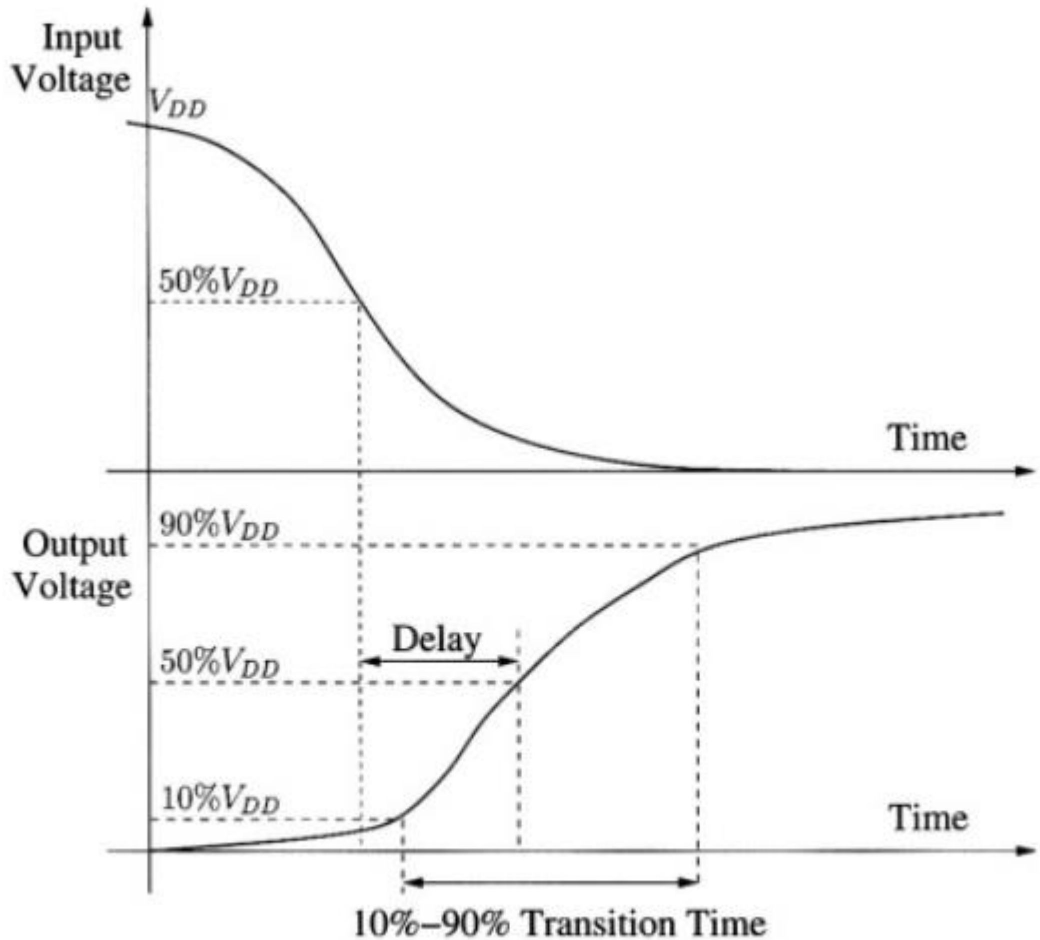
Delay

- The most fundamental way to compute delay is to develop a physical model of the circuit of interest, write a differential equation describing the output voltage as a function of input voltage and time, and solve the equation.

- The solution of the differential equation is called the ***transient response***, and the **delay** is the time when the output reaches *VDD* /2



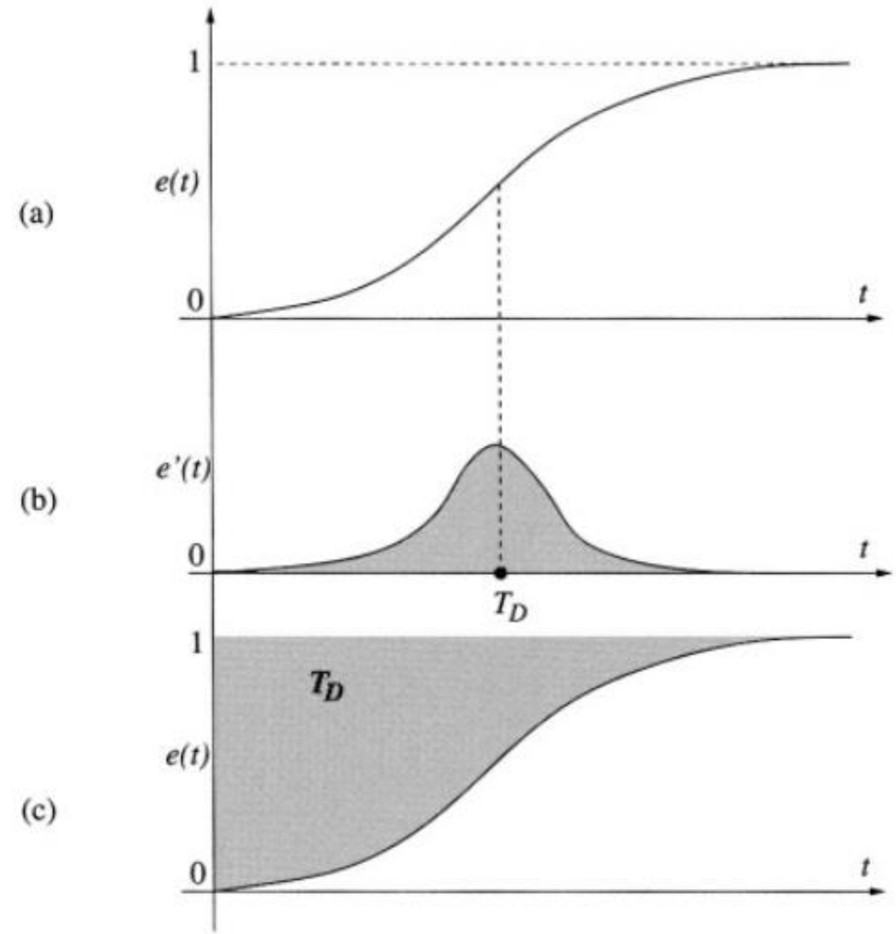
Elmore delay

The step response $e(t)$ at some node in a circuit (and its derivative $e'(t)$) is shown in left picture.

The time coordinate of the centroid of $e'(t)$ is the Elmore delay T_D

$$T_D = \int_0^\infty te'(t)dt$$

$$= \int_0^\infty [1 - e(t)]dt$$

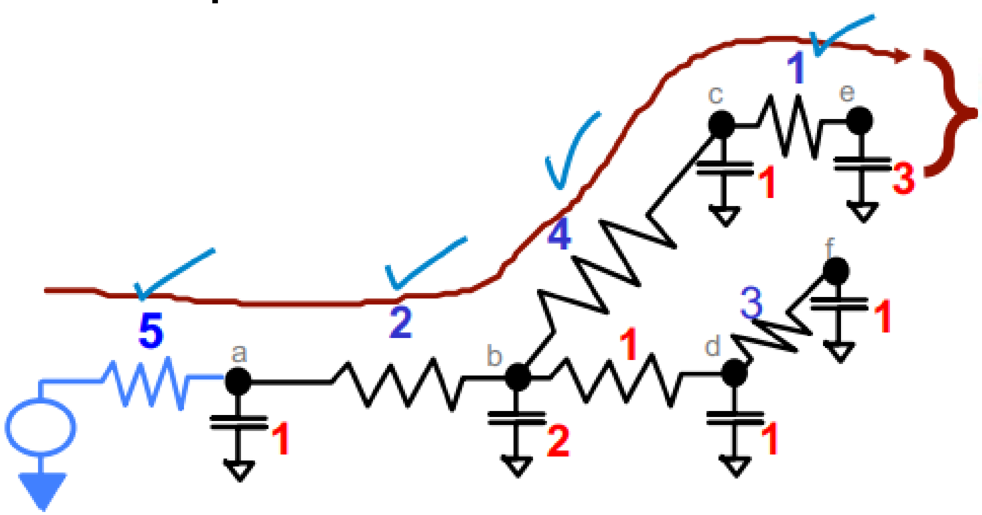


Elmore delay

The Elmore delay to node n_i in the RC tree is:

$$T_{Di} = \sum_{j \in P_i}^{n} R_j \sum_{k \in downstream(j)} C_k$$

Example:



$T_D(e)$ = 0
+5(1+2+1+1+3+1)
+2(2+1+1+3+1)
+4(1+3)
+1(3)
= 45+16+16+3
= 80

Asymptotic Waveform Evaluation

Moment of a transfer function

$$F(s) = \int_0^\infty f(t)e^{-st}dt$$ The Laplace transform of a transfer function

$$\begin{aligned} F(s) &= \int_0^\infty f(t)\left[1 - st + \frac{s^2t^2}{2!} - \frac{s^3t^3}{3!} + \cdots\right]dt \\ &= \int_0^\infty f(t)dt - s\int_0^\infty tf(t)dt + s^2\int_0^\infty \frac{t^2}{2!}f(t)dt - s^3\int_0^\infty \frac{t^3}{3!}f(t)dt + \cdots \\ &= m_0 + m_1 s + m_2 s^2 + m_3 s^3 + \cdots \end{aligned}$$

MacLaurin series expansion

$$\boldsymbol{F(s) = m_0 + \sum_i^\infty m_i s^i}$$

Moment matching

- **Pade's approximation:**

对于正整数m,n，函数f(x)在[m,n]阶的帕德逼近为

$$R(x) = \frac{\sum_{j=0}^{m} a_j x^j}{1+\sum_{k=1}^{n} b_k x^k} = \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m}{1 + b_1 x + b_2 x^2 + \cdots + b_n x^n}$$

且f(0)=R(0)，f'(0)=R'(0)，即 $f^n(0) = R^n(0)$

Let $$m_0 + m_1 s + m_2 s^2 + \cdots \equiv \frac{a_0 + a_1 s + \cdots + a_{q-1} s^{q-1}}{1 + b_1 s + \cdots + b_q s^q}$$

$$(m_0 + m_1 s + m_2 s^2 + \cdots)\cdot(1 + b_1 s + \cdots + b_q s^q) \equiv a_0 + a_1 s + \cdots + a_{q-1} s^{q-1}$$

$$m_0 + (m_1 + m_0 b_1)s + (m_2 + m_1 b_1)s^2 + \cdots \equiv a_0 + a_1 s + a_2 s^2 \cdots$$

$$\begin{array}{llcl}
s^0: & a_0 & = & m_0 \\
s^1: & a_1 & = & m_1 + m_0 b_1 \\
s^2: & a_2 & = & m_2 + m_1 b_1 + m_0 b_2 \\
 & & \vdots & \\
s^{q-1}: & a_{q-1} & = & m_{q-1} + m_{q-2} b_1 + \cdots m_1 b_{q-2} + m_0 b_{q-1}
\end{array}$$

MNA matrix

MNA matrix: $(\boldsymbol{G} + s\boldsymbol{C})\boldsymbol{X} = \boldsymbol{E}$

G / C: constant matrices (depend on the values of the RLC elements)

X: vector of unknowns

E: excitation vector

Represent **X** in terms of its moments

$$\mathbf{X} = \mathbf{m}_0 + \mathbf{m}_1 s + \mathbf{m}_2 s^2 + \mathbf{m}_3 s^3 + \cdots$$

When the excitation is $\delta(t)$, we have $\boldsymbol{E} = \boldsymbol{E}_{\mathbf{0}}$ in the s domain

$$(G + sC)(\mathbf{m}_0 + \mathbf{m}_1 s + \mathbf{m}_2 s^2 + \cdots) = \mathbf{E}_0$$

Moment computation

$$(G + sC)(\mathbf{m}_0 + \mathbf{m}_1 s + \mathbf{m}_2 s^2 + \cdots) = \mathbf{E}_0$$

$$\boldsymbol{Gm_0} + (\boldsymbol{Gm_1} + \boldsymbol{Cm_0})s + (\boldsymbol{Gm_2} + \boldsymbol{Cm_1})s^2 + \cdots = \boldsymbol{E_0}$$

$$\boldsymbol{Gm_0} = \boldsymbol{E_0} \quad (i = 0)$$

The solution of Equation is identical to that of the original system when an impulse excitation is applied, and is set **s** to zero.

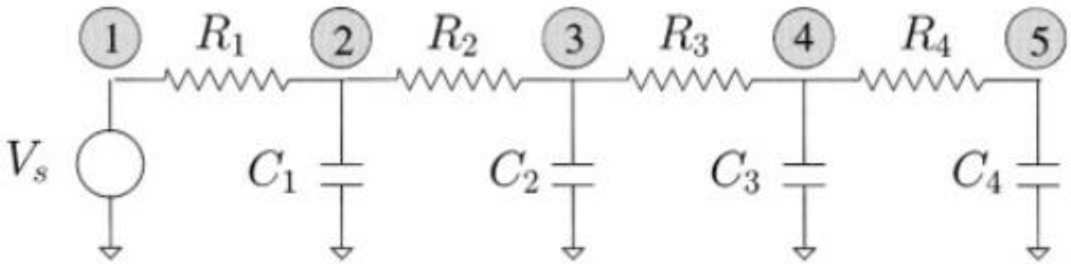
$$\boldsymbol{Gm_i} = -\boldsymbol{Cm_{i-1}} \quad (\forall\, i \geq 1)$$

This equation corresponds to the original excitation is set to zero, and a new excitation of $-\boldsymbol{Cm_{i-1}}$ is applied instead. Implies that the original circuit is modified as follows:

1. All voltage sources are short-circuited and current sources open-circuited.
2. Each capacitor is replaced by a current source of value $\boldsymbol{Cm_{i-1}}(\boldsymbol{V}_c)$, where $\mathbf{m_{i-1}}(\boldsymbol{V}_c)$ is the $(i-1)^{th}$ moment of the voltage $\boldsymbol{V}_c$ across the capacitor.
3. Each self or mutual inductance $\mathbf{L}_{ij}$ is replaced by a voltage source of value $\mathbf{L_{ij}}\boldsymbol{m_{i-1}}(\boldsymbol{I}_j)$ on line i, where $\mathbf{m}_{i-1}\left(\boldsymbol{I}_j\right)$ is the $(i-1)^{th}$ moment of the current through inductor j.

Example circuit

Moment calculation for a simple RC line



An example of an RC line.

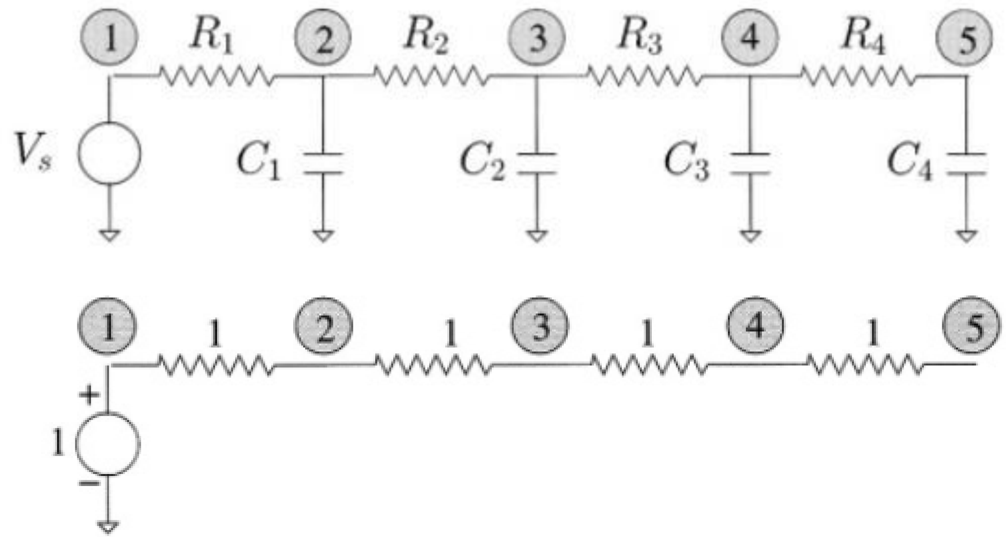
Let $R = 1\Omega, C = 1F$

$$m_q = [v_1\ v_2\ v_3\ v_4\ v_5\ i_v]^T$$

Example Step.1

- Finding m_0 i.e. $\boldsymbol{Gm_0} = \boldsymbol{E_0}$ $(i = 0)$

All capacitors are open-circuited, and the voltage source is replaced by a delta function in the time domain. (corresponds to a unit source in the $\boldsymbol{s}$ domain)



(a)

It is easily verified that no current can flow in the circuit.

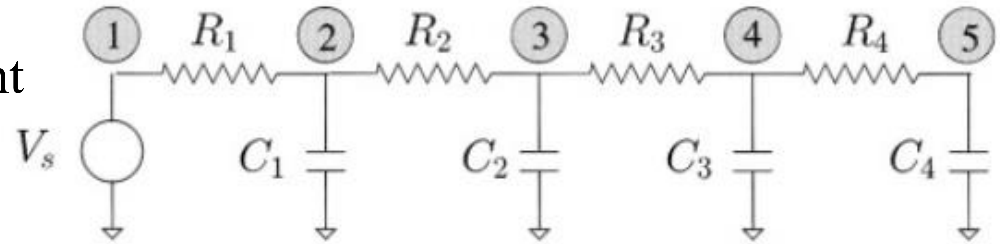
Therefore, the voltage at each node is 1.

$$\mathbf{m}_0 = [\ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0\ ]^T$$

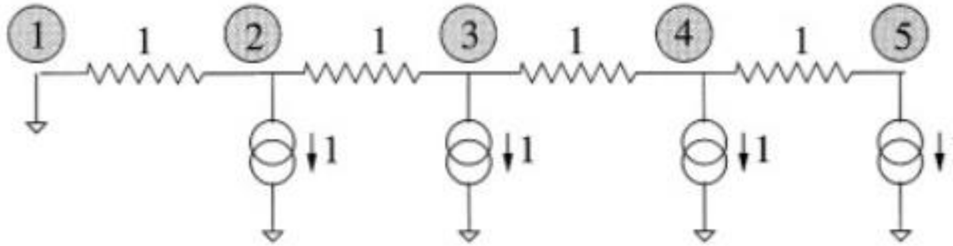
Example Step.2

- Finding m_1 i.e. $\boldsymbol{Gm_i} = -\boldsymbol{Cm_{i-1}}$ $(\forall\, i \geq 1)$

Capacitors is now replaced by a current source of value $\mathbf{C_i m_0 (V_i)}$



The current in each branch can be calculated as the **sum of all downstream currents**

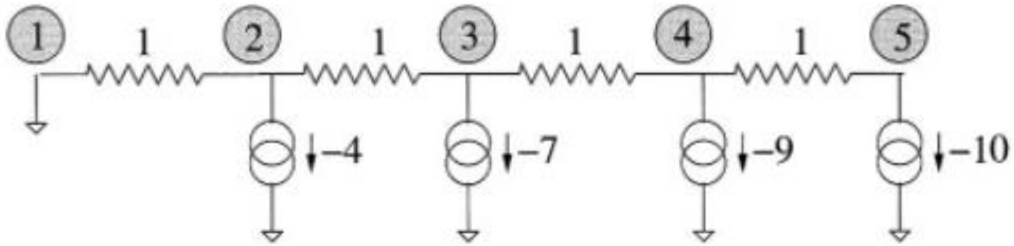


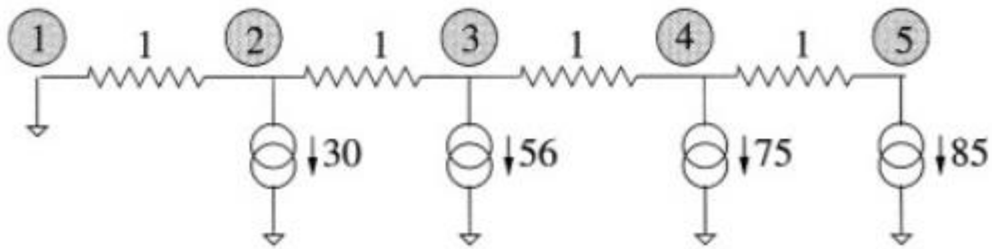
So the voltage at each node:

$$\mathbf{m}_1 = \begin{bmatrix} 0 & -4 & -7 & -9 & -10 & -4 \end{bmatrix}^T$$

Example Step.3

- Finding m_2, m_3





$$\mathbf{m}_2 = \begin{bmatrix} 0 & 30 & 56 & 75 & 85 & 30 \end{bmatrix}^T$$
$$\mathbf{m}_3 = \begin{bmatrix} 0 & -246 & -462 & -622 & -707 & -246 \end{bmatrix}^T$$

Example Step.4

	m_0	m_1	m_2	m_3
v_1	1	0	0	0
v_2	1	−4	30	−246
v_3	1	−7	56	−462
v_4	1	−9	75	−622
v_5	1	−10	85	−707
i_v	0	−4	30	−246

Pade's approximation:

$$
\begin{aligned}
s^0:&\quad a_0 = m_0\\
s^1:&\quad a_1 = m_1 + m_0 b_1\\
s^2:&\quad a_2 = m_2 + m_1 b_1 + m_0 b_2\\
s^3:&\quad a_3 = m_3 + m_2 b_1 + m_1 b_2 + m_0 b_3
\end{aligned}
$$

$$
\begin{aligned}
a_0 &= 1\\
a_1 &= b_1 - 10\\
0 &= b_2 - 10b_1 + 85\\
0 &= -10b_2 + 85b_1 - 707
\end{aligned}
$$

Example Step.4

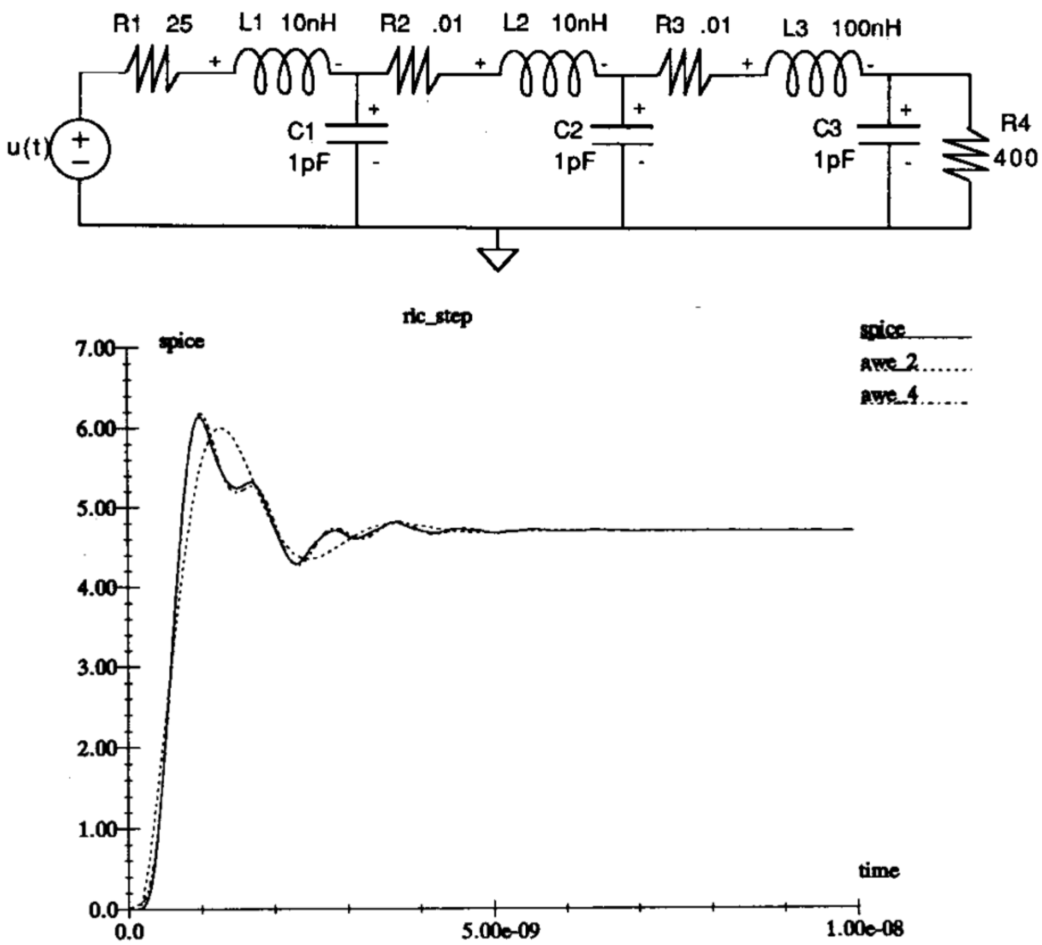
$$
\begin{aligned}
a_0 &= 1 \\
a_1 &= b_1 - 10 \\
0 &= b_2 - 10b_1 + 85 \\
0 &= -10b_2 + 85b_1 - 707
\end{aligned}
$$

$$
\begin{array}{cccc|c}
a_0 & a_1 & b_1 & b_2 & m \\
\hline
\multicolumn{4}{c|}{\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -10 & 1 \\ 0 & 0 & 85 & -10 \end{bmatrix}} & \begin{bmatrix} -1 \\ 10 \\ -85 \\ 707 \end{bmatrix}
\end{array}
$$

```
np.linalg.solve(mat,cons)
array([ 1.        , -0.46666667,  9.53333333, 10.33333333])
```

```
Latex(r"$H(s)=\frac{%d%.3fs}{1+%.3fs+%.3fs^2}$"%(a0,a1,b1,b2))
```

$$H(s) = \frac{1 - 0.467s}{1 + 9.533s + 10.333s^2}$$



L. W. Pillage and R. A. Rohrer, "Asymptotic waveform evaluation for timing analysis",

AWE second- and fourth-order approximations for the step response of a RLC circuit

Limitations of AWE

① The Pade approximations can be numerically unstable (higher moments becomes ill conditioned) .

- ***Pade via Lanczos process*** (improved numerical stability, but more computationally expensive)

  [Time: O(nmlog$n$), Space: O(m^2) for m Order]

② Can not guarantee passivity

- ***Passive Reduced-order Interconnect Macromodeling Algorithm*** (PRIMA).
- ***Arnoldi algorithm***
- ***Pole Analysis via Congruence Transformations*** (PACT)

  [Time: O($n^{1.5}$), Space: O($n$log$n$)]

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